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An improved acquaintance immunization strategy for complex network

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HIGHLIGHTS

- We create a new strategy to suppress epidemic on complex network.
- Our strategy takes time-varying and structure information into consideration.
- Our strategy is an improvement to acquaintance immunization strategy.

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ABSTRACT

The acquaintance immunization strategy is a common strategy to suppress epidemic on complex network which achieves a seemingly perfect balance between cost and effectiveness compared with other canonical immunization strategies. However, the acquaintance immunization strategy fails to take the time-varying factor and local information of nodes into consideration, which limits its effectiveness in some specific network topology. Our improved immunization strategy is based on a new mathematical model Network Structure Index (NSI), which digs deep to measure the connection property and surrounding influence of a node's neighbor nodes to better determine the importance of nodes during immunization. Both mathematical derivation and the simulation program tested on various network topology support our idea that this improved acquaintance immunization strategy protects more nodes from infection and immunizes important nodes more efficiently than the original strategies. As to say, our strategy has more adaptability and achieves a more reasonable balanced point between cost and effectiveness.

1. Introduction

Epidemic is an important issue to our lives. Both worm virus on Internet and Ebola virus disease spreading rampantly in Africa have caused great threat and panic to the masses. The suppression of epidemic attracts much attention in recent decades. Generally, there are several classic immunization strategies to suppress the epidemics on networks such as the random immunization (Anderson and May, 1992), the target immunization (Dobrescu, 2007), and the acquaintance immunization (Cohen et al., 2003). All of these strategies are conditioned by the immunization cost and immunization effectiveness, which are influenced by network topology, information of the network we have, possibility of virus spreading, size of the network, etc. Following are the obvious limitations of these three classic strategies.

Random immunization strategy immunizes a node randomly. It requires high immunization threshold which means it need immunize a very large fraction of a network to be effective. Target immunization strategy immunizes a node with most neighbor nodes. It is of great accuracy and effectiveness, but it is based on global information about the network, which is not available for most occasions. Acquaintance immunization strategy avoids the disadvantages of the previous strategies. It randomly chooses a node and randomly immunizes one of its neighbor nodes. Little information about networks is required, but randomly immunizing neighbor node is of blindness and is not efficient enough to protect important nodes, especially to some particular network topology.

As all these existing canonical strategies have their obvious limitations, we badly need an improved strategy which is more adaptive to almost all network topology and achieve a better balance between cost and effectiveness.

In recent years, many research works have shown up to present new ideas of finding a more effective and practical immunization strategy. Many of these methods are based on the acquaintance

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immunization, focusing on finding common neighbors (Pan Liu, 2009), searching out highly connected nodes to build a threaded-tree (Chai et al., 2011), double- immunization strategy (Jing et al., 2012), etc. These methods are effective in certain situation, however, they cannot avoid the limitation of the acquaintance immunization effectively. Other researchers focus on the importance ranking of nodes, which is to determine the ranking with a time-varying perspective (Starnini et al., 2013; Hill and Braha, 2010) or local information (Xin et al., 2011; Hadjichrysanthou and Sharkey, 2015), but these methods fail to deploy the benefits of acquaintance immunization strategy.

The immunization strategy based on acquaintance immunization provides a strong adaptability of various network structure using little information about the whole network; determining the importance of nodes with local and time-varying information ensures the accuracy during the immunization. So we propose a strategy combining these two benefits may be an innovative approach towards an effective and practical immunization strategy which can also achieve the balance between the amount of information we need and the result we can achieve.

In this paper, we start from a specific topology and propose the idea that the connection property and surrounding influence of nodes should not be neglected during immunization, which also reveals the limitation of the acquaintance immunization strategy. Based on that, we create a new index, Network Structure Index (NSI) to comprehensively assess the value, especially the potential value of nodes. With NSI which considers local and time-varying factor of a node, we improved the acquaintance immunization strategy by immunizing the neighbor node with highest NSI value. After that, mathematic derivation and computer simulation are used to support the advantage of our strategy.

2. Model

2.1. Possible improvements on acquaintance immunization

In many situations, it is so hard for us to acquire the global information about the network. Comparatively we can easily get the information of a selected node from the local network, especially some information of its nearby neighbors. In that case, target immunization is still not applicable, but acquaintance immunization can be greatly improved via changing the process of immunization with the help of its neighbor information and immunizing a most valuable neighbor instead of a random one.

Network is made up of many nodes and connections. We can easily notice that the importance of each node is diverse. Before we take actions to suppress the epidemics on networks, we need to find an objective value to give us reference about which node is worth protecting

The degree of node is a possible reference. The degree is the number of its links connected to other nodes. One solution is to immunize the neighbor node with highest degree. But the solution is not ideal in some occasions such as the case shown in Fig. 1.

Node 1 marked in red is the randomly selected node which is able to immunize a neighbor node. Its neighbor node 2 has a degree of four, while neighbor node 3 has a degree of two, but clearly we should immunize node 3 instead of node 2 because it has much more child nodes than node 2.

Another possible solution is to immunize the neighbor node with highest betweenness centrality. Betweenness centrality is equal to the number of shortest paths from all vertices to all others that pass through that node. It is an obvious and useful measure of both the load and importance of a node, but the calculation needs the knowledge of a large part of network topology, which is not easy to realize. As to say, if we have known the whole network topology, we would choose the target immunization instead.

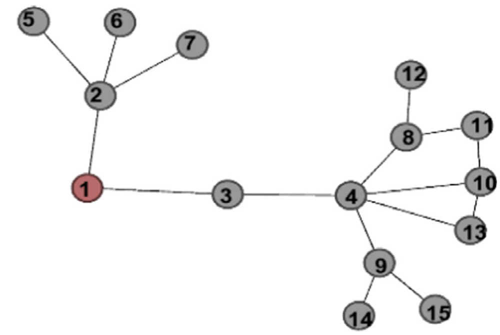

Fig. 1. One occasion when immunizing top degree neighbor node is not ideal.

So creating a new and low cost index to assess the value of each node is of top priority. The index should not only consider its degree, but also reflects its connection property and surrounding influence in its nearby sub-network as well.

We create such index called Network Structure Index (NSI).

2.2. Network Structure Index (NSI)

Suppose there is a network with m nodes. We can use an m∗m matrix to describe the connectivity of the network. We know that the connection properties and defense systems of each connection are not the same, so the possibility a node to be infected is not the same to each other. To have a clear view of that possibility, we define the p_{ij} which means node i has a percentage of p_{ij} spreading virus to node j. $p_{ij}=0$ means that node i and node j are not connected.

$$\text{The connectivity matrix } P=\begin{bmatrix} p_{11},p_{12},\ldots\ldots,p_{1M} \\ p_{21},p_{22},\ldots\ldots,p_{2M} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ p_{M1},p_{M2},\ldots\ldots,p_{MM} \end{bmatrix}$$

Each node has different values. For example, computers used in banks are much more important than computers used for entertainment at home, we define the value of node i as v_i to mark their difference. To be normalized, $v_i=1$ represents the nodes with highest importance $(0<i<M)$.

So when the node i is infected and starts spreading virus, the potential damage to node j is $d_{ij}=p_{ij}*v_j$. If node i and node j is connected directly, d_{ij} is a positive number, otherwise it is zero.

Before we look into the influence of a node in its nearby sub-network, we must have a clear view of nodes surrounding it and classify them into groups according to their importance. A node has neighbors which are the nodes connected directly with the node, it also has level 2 neighbors which are the neighbors of its neighbors, and so on. We can mark them as level 1–level T with the following steps. T is the number of top level we choose to stop the marking process.

Firstly, mark the node we choose to analyze (node 1 in Fig. 2) as level 1 (painted blue in Fig. 2).

Secondly, mark the nodes which have direct connection with level 1(node 2, 3, 4, 5) as level 2 (painted green in Fig. 2).

Thirdly, from each node of level 2, mark nodes which have direct connection with it (nodes 6, 7, 8, 9, 10 in Fig. 2) as level 3 (painted gray in Fig. 2). If the node has already been marked, ignore them (node 5 in Fig. 2).

Fourthly, mark higher level numbers from previous level nodes until the level T.

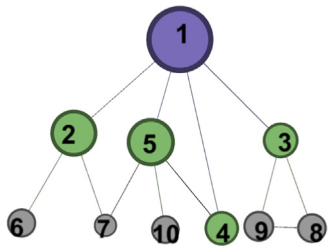

Fig. 2. An example of marking neighbor levels. The large blue node is the node we choose to analyze, the green and gray ones are its level 2 and level 3 neighbor nodes. Numbers in the circles are its IDs. Number of Top level in this graph is 3. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

After the classification, we can count numbers of connections between two adjacent levels as n_{ij}. The total potential damage through connections between level G nodes to level H nodes is

$$L_{GH} = \sum_i \sum_j d_{ij}$$

where i is all the nodes in level G, j is all the nodes in level H. $G, H < T$.

We can arrange them in a $T*T$ matrix N

$$N = \begin{bmatrix} L_{11} & L_{12} & X & & X & X & X \\ L_{21} & L_{22} & L_{23} & \cdots & X & X & X \\ X & L_{32} & L_{33} & & X & X & X \\ & \vdots & & \ddots & & \vdots & \\ X & X & X & & L_{(T-2)(T-2)} & L_{(T-2)(T-1)} & X \\ X & X & X & \cdots & L_{(T-1)(T-2)} & L_{(T-1)(T-1)} & L_{(T-1)T} \\ X & X & X & & X & L_{T(T-1)} & L_{TT} \end{bmatrix}$$

X in matrix N means that there is no possibility to have connections between these two levels, so X is always equal to zero.

When we view a sub-network, we care different levels with different emphasis. A manually controlled e_{GH} should be defined to present our perspective and show our emphasis. For example, in this virus spread issue, connections in level 1 and level 2 should be paid more attention than level 2 and level 3, so $e_{12} > e_{23}$. In order to be normalized, e is within the range of 0–1 as well.

$$E = \begin{bmatrix} L_{11}*e_{11} & L_{12}*e_{12} & X & & X & X & X \\ L_{21}*e_{21} & L_{22}*e_{22} & L_{23}*L_{23} & \cdots & X & X & X \\ X & L_{32}*e_{32} & L_{33}*e_{33} & & X & X & X \\ & \vdots & & \ddots & & \vdots & \\ X & X & X & & L_{(T-2)(T-2)}*e_{(T-2)(T-2)} & L_{(T-2)(T-1)}*e_{(T-2)(T-1)} & X \\ X & X & X & \cdots & L_{(T-1)(T-2)}*e_{(T-1)(T-2)} & L_{(T-1)(T-1)}*e_{(T-1)(T-1)} & L_{(T-1)T}*e_{(T-1)T} \\ X & X & X & & X & L_{T(T-1)}*e_{T(T-1)} & L_{TT}*e_{TT} \end{bmatrix}$$

X is always equal to zero.

We can find that all positive numbers are located in the narrow stripe in the diagonal, add them all, the result is the Network Structure Index (NSI).

$$NSI = L_{11}*e_{11} + L_{12}*e_{12} + L_{T(T-1)}*e_{T(T-1)} + L_{TT}*e_{TT} + \sum_{G=2}^{T-1} (L_{G(G-1)}*e_{G(G-1)} + L_{GG}*e_{GG} + L_{G(G+1)}*e_{G(G+1)})$$

NSI is a useful index for reference, which reveals the level structure, dimension, value of nearby nodes and vulnerability of a network. The high NSI means that the network structure is low in number of levels, large in scale, high in node value. So it is easy to spread fast and need special care and immunization as soon as possible.

2.3. In-depth acquaintance immunization

From Fig. 1, we know that attention should not only be paid to level 1 neighbor, but also higher number level nodes. NSI, with a node's connection property and influence in its nearby sub-network considered, is an ideal index to refer to. So we improve the acquaintance immunization strategy by immunizing the neighbor node with highest NSI value. As NSI is created by digging deep to find In-depth properties of a node, we call this improved strategy In-depth acquaintance immunization.

The complete procedure of In-depth acquaintance immunization is listed as below.

Step 1, randomly select a node from the network.
Step 2, find all its neighbor nodes which have not been immunized. If all its neighbor nodes have been immunized, restart the process from Step 1. If no node has neighbors, jump to Step 10.
Step 3, according to the information we can acquire from the chosen node and its nearby sub-network, define a connectivity matrix P to list all connection possibilities between nodes. If some connections are unable to find their connection possibilities, set them as average value of 0.5.
Step 4, define a suitable maximum level T for level classification. T should not be too small to waste information. And it should not be too large for there may not enough information and larger T means more time to calculate.
Step 5, define a set of suitable variables e_{GH} to represent our concern about different levels. In general, e_{GH} with smaller G and H should be defined a larger number.
Step 6, define a set of node values v_i to assess the node damage cost. If some neighbor nodes are unable to assess their value, set them as average value of 0.5.
Step 7, calculate NSI for every neighbor node.
Step 8, immunize the neighbor node m which has not been immunized with highest NSI value. Change column m in P matrix to 0, meaning no nodes can connect to node m forever.
Step 9, choose the immunized node as the next selected node. Repeat the process from step 2.
Step 10, from node 1 to node M, immunize nodes which have not been immunized one at a time.

Fig. 1 is shown as an example.

Suppose node 1 is the randomly selected node. It has two unimmunized neighbor nodes 2 and 3. We know most of its neighbor connection possibilities, but possibilities of some connections (8–11, 8–12 and 10–13) are unknown, so $p_{8(11)}, p_{8(12)}, p_{(10)(13)} = 0.5$.

We find that many nodes existed as level 4 nodes under node 3, so we can define T as 4.

$$
\begin{aligned}
P = [&0.00, 0.58, 0.61, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.31, 0.00, 0.00, 0.00, 0.90, 0.74, 0.62, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.57, 0.00, 0.00, 0.54, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.00, 0.13, 0.00, 0.00, 0.00, 0.00, 0.74, 0.85, 0.53, 0.00, 0.00, 0.37, 0.00, 0.00;\\
&0.00, 0.98, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.78, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.70, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.00, 0.00, 0.44, 0.00, 0.00, 0.00, 0.00, 0.40, 0.00, 0.50, 0.50, 0.00, 0.00, 0.00;\\
&0.00, 0.00, 0.00, 0.80, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.63, 0.41;\\
&0.00, 0.00, 0.00, 0.61, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.16, 0.00, 0.50, 0.00, 0.00;\\
&0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.92, 0.00, 0.36, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.38, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.00, 0.00, 0.18, 0.00, 0.00, 0.00, 0.00, 0.00, 0.58, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.13, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00;\\
&0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.7, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00]
\end{aligned}
$$

A suitable set of e_{GH} is listed for reference.

$$
\begin{aligned}
e_{GH(4)} = [&0.0, 1.0, 0.0, 0.0;\\
&1.0, 0.6, 0.5, 0.0;\\
&0.0, 0.5, 0.3, 0.2;\\
&0.0, 0.0, 0.2, 0.1;]
\end{aligned}
$$

$$v_i = [1.0, 0.6, 0.6, 0.3, 0.5, 0.7, 0.8, 0.5, 0.3, 0.3, 0.7, 1.0, 0.3, 0.2, 0.4]$$

Node 5 and 8 are unable to assess their value, so v_5 and $v_8 = 0.5$.

According to NSI algorithm, neighbor node 2 has an NSI of 2.914, while neighbor node 3 has a larger NSI of 3.586, so we choose node 3 to immunize. p_{13} and $p_{43} = 0$.

Then node 3 becomes the next selected node which is able to immunize one of its neighbor nodes. The network level becomes lower, so there is no need to choose T as 4, we define T as 3 this time.

$$
\begin{aligned}
e_{GH(3)} = [&0.0, 1.0, 0.0;\\
&1.0, 0.6, 0.5,\\
&0.0, 0.5, 0.3]
\end{aligned}
$$

Its neighbor node 1 has an NSI of 1.483, while neighbor node 4 has a larger NSI of 2.8279, so we choose node 4 to immunize this time. Set p_{34}, p_{84}, p_{94}, $p_{(10)4}$, $p_{(13)4} = 0$.

Then node 4 becomes the next selected node. Its unimmunized neighbor nodes 8, 9, 10, 13 have NSI of 1.61, 0.539, 0.949, 0.434 respectively. So the immunization should be applied to node 8.

From these three immunizations, we can find that every node immunized is of great importance under such circumstances. These immunizations have effectively disconnected these hub nodes or key nodes in the sub-network, greatly reducing the possibility of epidemic spread in the sub-network.

We can also find the benefit of selecting nodes previously immunized as the start of next round immunization is that hub nodes in deeper layer can be traced automatically. Node 4 in Fig. 1 is a large hub in level 3 from the perspective of node 1, but its potential value is presented by NSI of node 3, whose N_{GH} between level 2 and level 3 is 1.08, and N_{GH} between level 3 and level 4 is 0.78, taking up a percentage of 51.9% in its NSI, greatly enhancing that value, enabling NSI of node 3 greater than node 2 and veering the immunization direction towards node 4.

2.4. Mathematical justification

Scale-free network is a classic network topology and we can change the spread rate of virus to let it be more close to the real network. According to the characteristic of scale-free and continuous growth of real network, the virus spread tends to grow into a stable state and we can find the threshold value.

As we mentioned before, our method is influenced by two uncertain parameter d_{ij} and e_{GH}. So we cannot verify its effectiveness directly and need another conclusion as a bridge to get the conclusion. The conclusion is if we choose the nth central point among $(n-1)$th central point's neighbor instead of choosing it randomly, more nodes in the network will be immunized during a certain period.

The following are the analysis and certification. First, we need define the concept 'layer' in advance. The layer of a particular point means the shortest hop distance from the starting point which can be likely to be immunized (we need update the layer information to avoid possible loop). We assume there are $n_l(k')$ nodes of degree k' in some particular layer l which means that excluding the node we pass through, there are only $k-1$ new nodes in layer $l+1$. At the same time, we denote the event that a k-degree node is access to the disease as s_k. To compute the number of susceptible nodes in layer $l+1$, we have the following equation:

$$n_{l+1}(k) = p_b \sum_{k'} n_l(k')(k'-1)p(k|k', s_{k'})p(s_k|k, k', s_{k'})$$

where P_b is the probability of each directed link occupied. According to the early assumption, we choose the next immunization central among the neighbor of the last immunization central with probability $1/k''$. The probability of choosing one of its neighbor is $1/k$. After immunizing Np times, we can get the following inequation:

$$Vp(k') = (1 - \frac{1}{k'k''})^{Np} < (1 - \frac{1}{Nk'})^{Np} \approx e^{-p/k'} = (Vp(k'))_a$$

where $V_p(k')$ is the value of the acquaintance which is not selected in the Np times attempts according to the new principle and $(V_p(k'))_a$ represent the value of the acquaintance which is not selected in Np trials with the acquaintance immunization.

According to Bayles' rule, we get the following two equations to simplify $n_{l+1}(k)$

$$p(k|k', s_{k'}) = \frac{p(s_{k'}|k, k')p(k|k')}{p(s_{k'}|k')}$$

$$
\begin{aligned}
p(k|k', s_{k'}) &= \frac{\varphi(k)e^{-p/k} < e^{-p/k'} >^{k'-1}}{< e^{-p/k'} >^{k'}}\\
&= \frac{\varphi(k)e^{-p/k}}{< e^{-p/k'} >}
\end{aligned}
$$

where $\varphi(k) \equiv p(k|k') = kp(k)/ < k >$

Based on the another equation

$$p(s_k|k,k') = p(s_k|k,k',s_{k'}) \\ = e^{-p/k'} < e^{-p/k} >^{k-1}$$

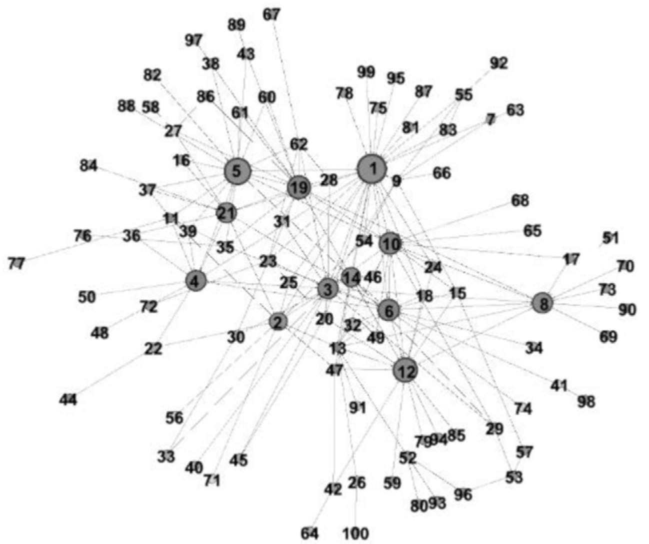

Fig. 3. A sample of Internet model with 100 nodes and 211 edges with no double edges generated by GDTANG. (With node labels).

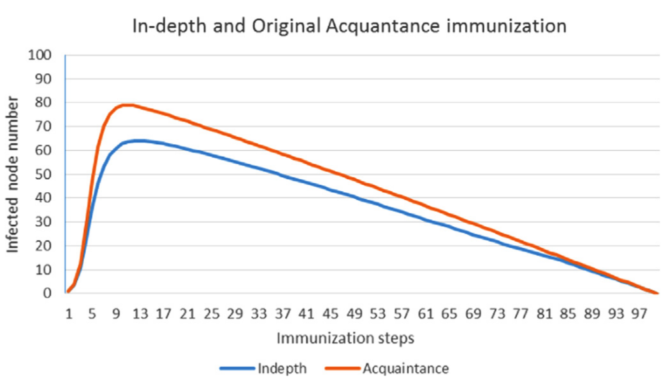

Fig. 4. The average number of 200 times simulations of infected node number during 100 immunization steps using In-depth Acquaintance Immunization strategy and Original one.

we can get

$$n_{l+1}(k) = p_b v_p^{k-2} \varphi(k) e^{-p/k} \sum_{k'} n_l(k')(k'-1) e^{-p/k'}$$

$$= n_l(k) p_b \sum_{k'} \varphi(k')(k'-1) v_p^{k'-2} e^{-2p/k'}$$

When $Np \to \infty$, the whole network tends to be stable and we can get the critical condition equation

$$n_{l+1}(k) = n_l(k)$$

So we can get p_c from the following equation:

$$p_b^{-1} = \sum_k \varphi(k)(k-1) v_{p_c}^{k-2} e^{-2p_c/k} (*)$$

where $p_b \to 1$ corresponds to full immunization and we can easily get the following important in Eq.:

$$f_c = 1 - \sum_k p(k) p(s_k|k) = 1 - \sum_k p(k) v_{p_c}^k$$

$$> 1 - \sum_k p(k) (v_{p_c}^k)_a$$

where f_c is the number of nodes immunized in N trials

From this in equation we can get that choosing the nth central point among $(n-1)$th central point's neighbor is much more effective than choosing the central point randomly in the acquaintance immunization.

Our In-depth immunization method is based on this conclusion. That means it have been much more effective than acquaintance immunization. Furthermore, we take the nodes value into consideration in our method to be more close to real world. Because we immunize one of the neighbor nodes which is of the most value, the strategy can be considered as making use of the advantage of target immunization in local world, which makes our strategy much more useful than the intermediate strategy, which immunizes a random neighbor of a random node. Taken the intermediate immunization strategy as a bridge. We can conclude that the intermediate immunization strategy is more effective than classic acquaintance immunization, but performs worse than our In-depth acquaintance immunization.

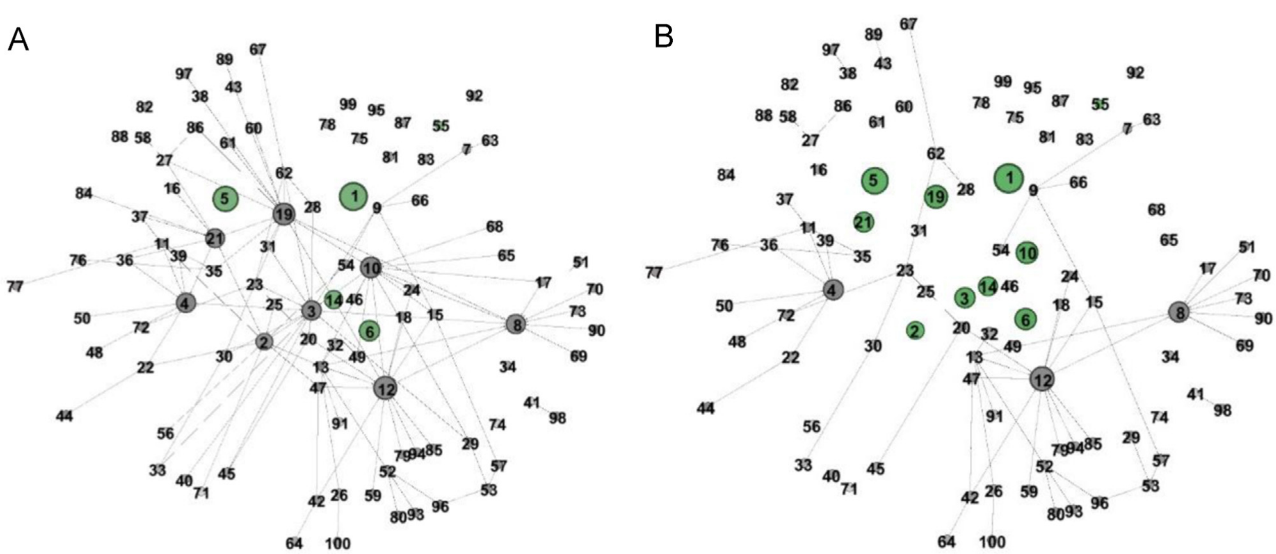

Fig. 5. The network topology after cutting connections from 5(A) and 10(B) immunized nodes under the guidance of In-depth Acquaintance Immunization.

3. Result

3.1. Computer simulation

In order to reveal a visual comparison between different strategies, we simulate our strategy in a relatively real network. We use GDTANG (The Geographic Directed Tel Aviv University Network Generator) (Bar et al., 2005), a Perl program to generate synthetic Internet-like topologies using an improved BA type model. It produces networks with a power-law degree distribution, realistic maximal degrees and "Dense Core", accurate number of leaves and geographically meaningful clusters, which is an ideal network generator.

To clearly illustrate our method, we created an Internet model with 100 nodes and no double edges. Other network generating parameters use default values. Importing data to Gephi, a network analysis software, after some location adjustment, the network shows in Fig. 3.

The simulation is based on the following assumptions:

1. Infection or immunization has steps of time. Each step of time, a random node is infected, and we can immunize (recover) one node (using different methods to choose which node should be immunized).
2. Immunized nodes cannot recover its neighbor nodes. But each infected node has a chance of p_{ij} to spread virus to its neighbor nodes (node i is the infected node, node j is its neighbor node). If not succeed, the attack will be continued in the next step of time until the target is infected or immunized.
3. An immunized (recovered) node cannot be infected again.
4. Connections in the simulated network have directions.

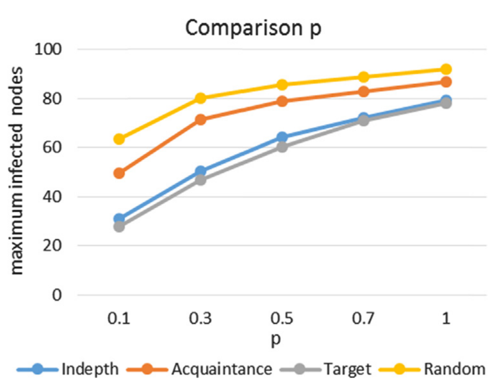

Fig. 6. The fluctuations of numbers of infected nodes during 100 steps of time using four Immunization strategy (In-depth Acquaintance Immunization, Acquaintance Immunization, Target Immunization and Random Immunization) on GDTANG network with various connection possibilities (p).

Under these four assumptions, we program our In-depth acquaintance immunization strategy and three common immunization strategies, the random immunization, the target immunization and the acquaintance immunization.

To be generalized, we define every $p=0.5$, every $v=1$ in our program. According to its network level, we choose $T=3$, and use $e_{GH(3)}$ defined in Section 2.3. Considering the uncertainty of random selection in the program, we repeated the simulation for 200 times, and the In-depth acquaintance immunization results as well as the original acquaintance immunization results are shown in Fig. 4.

The results show that the In-depth acquaintance immunization strategy has average maximum infected node numbers of 64.135, while 79.030 in the original strategy, preventing 14.895 more nodes in the network from being infected. In addition, it suppresses the epidemic spread more efficiently. From Fig. 4, in In-depth acquaintance immunization, the increase rate of newly infected nodes in each step is lower than the original one, so the infected nodes number has become steady earlier.

We can have a clearer view of immunization process through a sample immunization in the network.

In this sample immunization process, the first ten immunized nodes of the In-depth acquaintance immunization are node 55, node 1, node 6, node 14, node 5, node 10, node 3, node2, node 21 and node 19 (in Fig. 3). Node 55 is only a marginal node with a degree of 4, but soon the strategy has traced top degree node 1 and other important hub nodes which are critical to the suppression of epidemic. After these ten steps of immunization, the connections in the center area of the network has become sparse (shown in Fig. 5), and the spread is under control.

3.2. More simulations and comparisons

After proving that the In-depth acquaintance immunization strategy is more effective than the original one on GDTANG network, we continue to research on other network properties that may influence its effect such as connection possibilities (p), network scale (n), and different network topologies. We also compare it with Target immunization and Random immunization strategy in each simulation.

3.2.1. GDTANG network with various connection possibilities (p)

In the previous simulation, $p=0.5$ to better reflect comparison caused by varied immunization strategies. We change p from 0.1 to 1 to further research its influence to the results. Results using four strategies (In-depth Acquaintance Immunization, Acquaintance Immunization, Target Immunization and Random Immunization) are shown in Fig. 6 in the same line chart. Every simulation result is the average number of 200 repeated times.

The general situations are similar to the previous simulation, which are, target immunization performs best, In-depth acquaintance

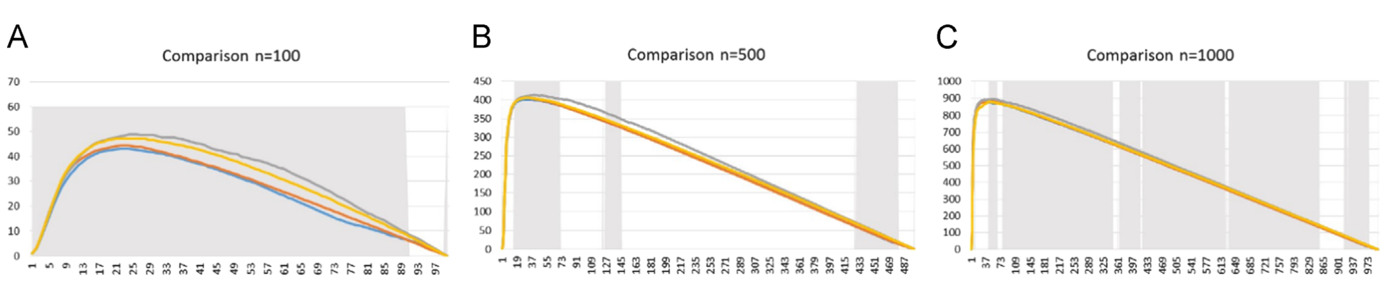

Fig. 7. The fluctuations of numbers of infected nodes during 100 steps of time using four Immunization strategy (In-depth Acquaintance Immunization, Acquaintance Immunization, Target Immunization and Random Immunization) on Random graph with various scale. Shadow area is the period of time when In-depth acquaintance immunization performs best.

immunization closely follows, while other two are left behind. In-depth immunization strategy is much better than the original acquaintance immunization.

We can conclude that, the lower p, the harder a virus pass to its neighbor and the lower maximum number of nodes infected. The lower p also means that there is more time for the immunization to be established, and after that, virus is easily controlled, so when $p=0.1$, the differences between best and worst immunization strategy are most evident.

3.2.2. *Random graph with various scales*

We further researched on other network topologies and considered the impact of scale (node numbers n).

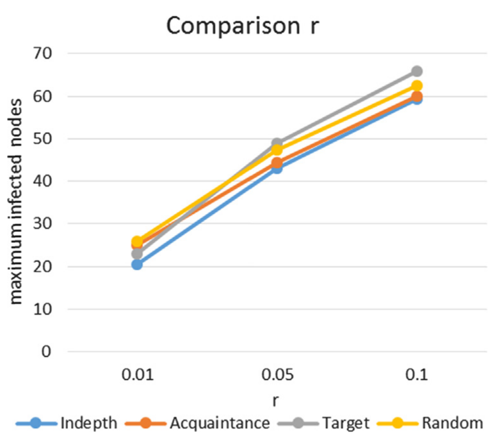

Fig. 8. The fluctuations of numbers of infected nodes during 100 steps of time using four Immunization strategy (In-depth Acquaintance Immunization, Acquaintance Immunization, Target Immunization and Random Immunization) on Random graph with various wiring probabilities.

We generated directed random graph of 0.05 wiring probability and 0.5 spread probability by Gephi. Network scale varied from 100 to 1000. Results are shown in Fig. 7.

In this case, due to its randomness, single node can hardly own large degree, so target immunization is not perfect. While In-depth acquaintance immunization still performs well, in most cases it is the best immunization strategy as shown in the shadow area in Fig. 7. The In-depth acquaintance immunization strategy has a character of localized target immunization as is mentioned before, which is clearly demonstrated in this set of simulation. It calculates NSI of a node according to its neighbors, which has much stronger adaptability than other strategies.

3.2.3. *Random graph with various wiring probabilities (r)*

We further research on the density of connections in a network, changing wiring probability (r) from 0.01 to 0.1. Node number $n=100$, spread possibility $p=0.5$. Results are shown in Fig. 8.

The increased r means more connections between nodes in a network. As the connections increase, the degree of nodes increase as well, which causes many nodes with high degree, so the target immunization becomes the least effective when $r=0.1$. But that case does not happen in both original and In-depth acquaintance immunizations, and the In-depth one is perfect when r is small as well. The density of connections may cause great differences to other strategies, but to In-depth acquaintance immunization, its impact is minimized due to its character of localized target immunization.

3.2.4. *WS-Small World Model with various replacement probabilities (r)*

WS-Small World model is another important network topology. We tested four strategies with various replacement probability (r) from 0.001 to 0.5.

Node number $n=500$. Number of neighbors on each side of a vertex is 2. Results are shown in Fig. 9.

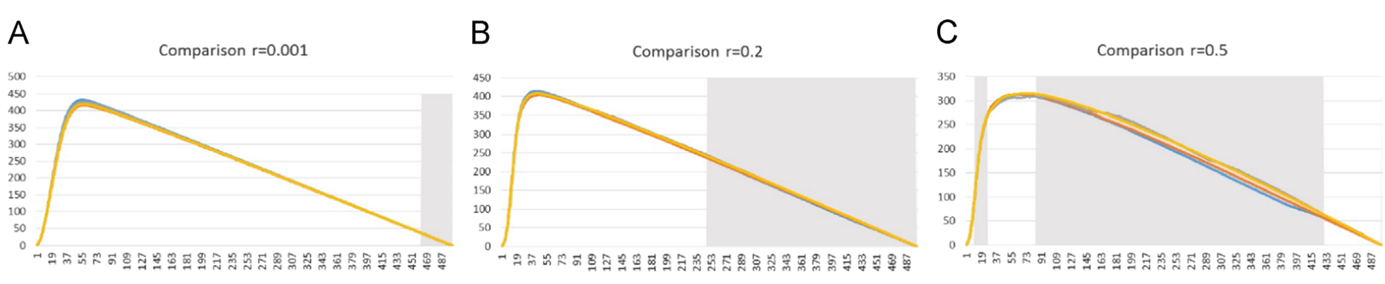

Fig. 9. The fluctuations of numbers of infected nodes during 100 steps of time using four Immunization strategy (In-depth Acquaintance Immunization, Acquaintance Immunization, Target Immunization and Random Immunization) on WS-Small World Model with various replacement probability (r). Shadow area is the period of time when In-depth acquaintance immunization performs best.

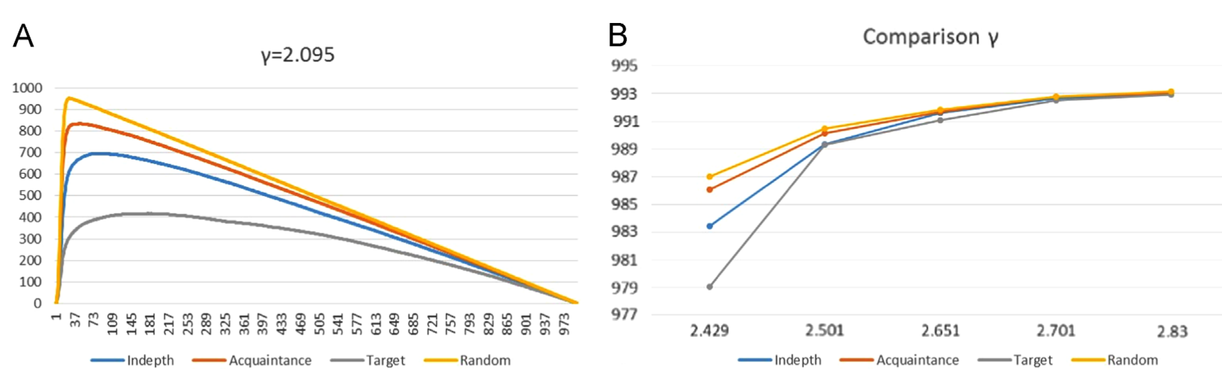

Fig. 10. The fluctuations of numbers of infected nodes during 100 steps of time using four Immunization strategy (In-depth Acquaintance Immunization, Acquaintance Immunization, Target Immunization and Random Immunization) on Scale free model with various exponents of the degree distribution (γ).

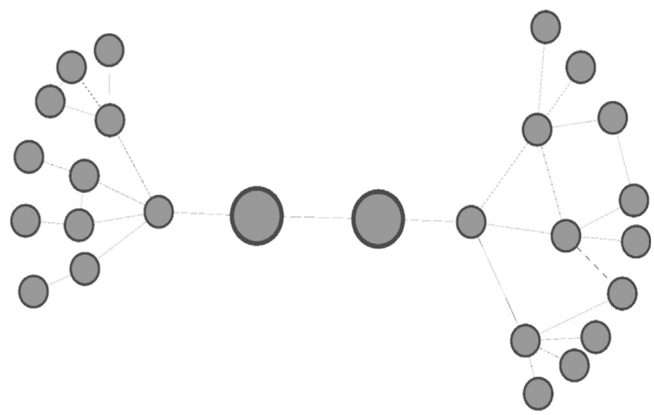

Fig. 11. An extreme case of two sub-network chained with a line of nodes (two bigger nodes in the middle are not easy to be immunized using existing strategies).

The increasing r means that the network is more irregular. When r is close to 0, network is a regular lattice, so In-depth acquaintance immunization strategy immunizes nodes one after another by its neighbor, which is caused by its continuous immunization character for the convenience of potential value nodes tracing. In this case, it becomes an obstacle. For random immunization cuts lattice into random pieces, preventing virus spreading to other pieces, so it becomes the most effective strategy, but In-depth acquaintance immunization is unable to do that. It only immunizes nodes from a certain part of network, failing to immunize other parts. But while network is more irregular, such case no longer exists.

So regular graph becomes the Achilles heel to In-depth immunization strategy.

3.2.5. *Scale free model with various exponents of the degree distribution (γ)*

We generate directed Scale free model with the following properties by scale free network generator in matlab. Number of vertices:1000; *exponent of the degree distribution*: d from 2.095 to 2.83. Results are shown in Fig. 10.

The larger γ means denser in a scale free network. From Fig. 10, we find that in a sparse scale free network, the Target and In-depth acquaintance immunization is still perform well, but there is little difference between these four strategies when γ is greater than 2.501. When γ=2.83, the maximum infected node number is 992.96 (In-depth), 993.04 (Acquaintance), 992.92 (Target) and 993.14 (Random), the best strategy (Target) is only 0.22 better than worst (Random). In fact, towards so large and dense network, any strategy is ineffective due to the rapid spread of virus, so it is with little meaning to suppress virus in such a network.

4. Conclusion

The In-depth acquaintance immunization strategy is an improvement on the original acquaintance immunization strategy. The introduction of NSI (Network Structure Index) is of great practical value, which ideally reflects the connection property and surrounding influence of a node, helping our strategy to decide and immunize a neighbor node with the greatest importance. This immunization strategy do not need to know the global information about the network like target immunization, but by the information of nearby nodes, it proves to be more efficient in finding and immunizing important neighbor nodes.

In our simulation, we made these following conclusion. In GDTang and Scale-free network models, its performance is only second to target immunization; in most random graph, it is even the best strategy; while in WS-Small World Model, it is not effective when network is highly regular. But, in general, it has obvious advantage over the original acquaintance strategy. This improved immunization strategy protects more nodes from getting infected and has stronger ability to control spread rate.

In addition, its characters of localized target immunization and continuous immunization enable it to adapt to different network topology better and have the capability of finding large sub-network and trace important nodes, especially when there are many nodes which have low-degree but high betweenness centrality in a network (Fig. 11). The unimmunized chain nodes are highly possible to pass epidemic from one sub-network to another, causing great damage. In this particular situation, random immunization is still blind, acquaintance immunization has little chance to immunize the chain nodes, and target immunization is the last to consider immunizing the chain nodes.

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